Diverse, Orthogonal Waveforms and Signal Processing Architecture for Joint GMTI and SAR Applications

Uttam K. Majumder, Mark R. Bell
School of Electrical and Computer Engineering
Purdue University, West Lafayette, Indiana 47907
Email: umajumde, mrb@purdue.edu

Muralidhar Rangaswamy
AFRL Sensors Directorate
WPAFB, Dayton, OH 45433

***Abstract*—In this research, we introduce a signal processing framework for joint GMTI and SAR algorithms that is based on orthogonal (transmit and receive) waveforms. Traditionally, radar systems are configured to operate either in GMTI or SAR processing mode, but not both simultaneously. This is due to the fact that operational parameters for these two modes are quite different. For example, exoclutter GMTI processing requires a high pulse repetition frequency (PRF), but a high PRF results in increased range ambiguity—and an increased processing burden—in SAR imaging. We propose combining diverse, orthogonal waveforms and introducing corresponding processing techniques to reduce the problems and complexities of joint GMTI and SAR exploitation. For the exoclutter GMTI problem, the necessary high-PRF pulse train will be used to achieve finer Doppler resolution for detecting fast moving objects. For the endoclutter GMTI and SAR imaging problem, we will transmit low PRF pulses. The goal for low PRF pulses for endoclutter GMTI and SAR imaging is to ensure that range ambiguity issue has been addressed. These new approaches will achieve following benefits: (1) accomplish GMTI and SAR processing concurrently by eliminating the complexities associated with reconfiguring a radar system, (2) more efficiently use bandwidth by employing appropriate bandwidth for exoclutter GMTI pulses and SAR image formation pulses, and (3) reduce range ambiguity issue associated with high PRF operation.**

***Index Terms*—Synthetic Aperture Radar(SAR), Ground Moving Target Indication (GMTI), Exoclutter GMTI, Endoclutter GMTI, Orthogonal Waveforms**

I. INTRODUCTION

Synthetic aperture radar (SAR), invented by Carl A. Wiley at Goodyear aircraft company, is a mode of radar operation that can interrogate a large ground swath and provide high-resolution images of the illuminated scene. SAR has had huge impact both in both civilian and military applications. In the civilian sector, SAR applications include ground mappings for disaster planning, remote sensing for vegetation and crop information, planetary exploration (e.g., the NASA Venus Radar Mapper), oil and mineral exploration, and medical imaging. In the military sector, SAR technology has been used for high resolution imaging surveillance. In SAR-based technology development, computation plays a vital role. In recent years, computations became very cost effective. Hence, SAR technologies have been evolving and benefitting both civilian and military applications such as ground mapping by NASA for topographical information.

SAR is a mature technology and hence many well written books and journal articles have been published describing its operation. The book by Curlander and McDonough[2] provides a good overview of basic SAR theory and systems, while the monograph by Soumekh [3] provides a comprehensive treatment of SAR signal processing.

In this paper, our objective is to establish an efficient signal processing architecture for joint GMTI and SAR measurement and processing. It will provide us the ability to simultaneously detect moving objects (both fast and slow movers) and generate SAR images of the ground. When an object on the ground is large and moves fast, it generates a large Doppler shift, and the form of GMTI processing used to detect this type of object is known as exoclutter GMTI. With appropriate calibration and parameters settings, one can apply range-Doppler processing to detect this object. However, if a ground object is relatively small (in a heavily clutter environment) and moves slowly, its Doppler is small. The form of GMTI processing used to detect this type of object is known as endoclutter GMTI. When trying to detect slow moving targets, standard range-Doppler processing may not work. Advanced techniques such as space-time adaptive processing (STAP), displaced phase center antenna (DPCA) based coherent change detection (CCD) must then be used to detect slow moving objects in a high clutter environment. For slow moving target detection, STAP algorithm may not provide good results [17]. In this research, we will present coherent change detection to detect slow movers. This paper has been organized into following manner: (a) A short literature review on joint GMTI and SAR research, (b) diverse, orthogonal waveform design, (c) Our technical approaches for joint GMTI and SAR processing, (d) Results, (e) Discussion, and (e) Conclusion.

II. A SHORT LITERATURE REVIEW ON JOINT GMTI AND SAR RESEARCH

As an important research problem, joint GMTI and SAR processing engaged researchers from both the defense industry [7], [16], [14], and [13] and academia [12]. Many researchers have attempted to solve only the endoclutter GMTI problem (i.e. change detection-based GMTI or STAP-based GMTI). Murthy, Pillai, Davis [17] presented frequency-jump burst waveforms for simultaneous SAR and GMTI. Davis[18] also presented a common waveform for simultaneous SAR and GMTI. As mentioned earlier, if an object moves slowly and accurate registration of two time-successive SAR images can be performed, endoclutter GMTI provides good detection performance of the slow moving object (assuming range ambiguity is not an issue). However, for faster-moving ground objects, the exoclutter GMTI method will be needed. Consider an operating environment where we can expect both slow and fast moving objects. In this scenario, we may have to run both exoclutter and endoclutter GMTI algorithms. However, PRF requirements for these two methods are different (exoclutter GMTI will require a higher PRF than endoclutter GMTI). Hence, one possible strategy would be to design a radar system to transmit some pulses with high PRF (to detect fast movers) and then transmit some pulses with low PRF to detect slow mover and SAR image formation. Our proposed approach implements above strategy. Further, our approach can provide additional capabilities such as efficient bandwidth utilization for SAR pulses (higher bandwidth) and GMTI pulses (lower bandwidth).

The merits of using diverse waveforms for imaging has been studied in the past [1]. The paper by Bell and Monrocq [5] outlined a multiplexed-waveform Doppler filter bank concept for diverse, orthogonal waveforms. Majumder, Bell, and Ranagaswamy [4] presented LFM-based Doppler tolerant, orthogonal waveform design. Among other papers, these papers motivated our approach to develop a joint GMTI and SAR signal processing architecture.

III. Diverse, Orthogonal Waveform Design

Diverse, orthogonal waveforms are vital for our approach to joint GMTI and SAR processing. In our previous work [4], we presented LFM-based diverse, orthogonal waveforms design applying direct-sequence spread-spectrum coding technique to LFM waveforms. In this section, we briefly show closed-form mathematical expression of our waveform's ambiguity function.

Let

$$s(t) = e^{i\pi\alpha t^2}\cdot 1_{[0,T]}(t)$$

be an LFM waveform with LFM index α and duration T, where

$$1_{[0,T]}(t) = \begin{cases} 1, & 0\leq t\leq T, \\ 0, & \text{otherwise.} \end{cases} \qquad (1)$$

is the indicator function of the time interval $[0,T]$. Now define two direct sequence spread-spectrum coded LFM signals

$$s_1(t) = \sum_{m}^{M-1} C_m p(t-mT_c)e^{i\pi\alpha_1 t^2}, \qquad (2)$$

and

$$s_2(t) = \sum_{n}^{M-1} D_n p(t-nT_c)e^{i\pi\alpha_2 t^2}, \qquad (3)$$

where

C_m : first code sequence,
D_n : second code sequence (different from C_m,)
T_c : chip time,
$p(t)$: rectangular pulse,
$\alpha_1,\ \alpha_2$: different chirp rates.

Then the cross-ambiguity function of $s_1(t)$ and $s_2(t)$, which is defined as

$$\chi_{s_1,s_2}(\tau,\nu) = \int_R s_1(t)s_2^*(t-\tau)e^{i2\pi\nu t}\,dt$$

then becomes

$$\begin{aligned}\chi_{s_1,s_2}(\tau,\nu) &= \int_R \left(\sum_{m=0}^{M-1} C_m p(t-mT_C)\cdot e^{i\pi\alpha_1 t^2}\right) \\ &\quad \times \left(\sum_{n=0}^{M-1} D_n p(t-nT_c-\tau).e^{i\pi\alpha_2(t-\tau)^2}\right)^* e^{i2\pi\nu t}\,dt \\ &= \sum_{m=0}^{M-1}\sum_{n=0}^{M-1} C_m D_n^*\cdot f(m,n,\tau,\nu),\end{aligned}$$

where

$$\begin{aligned} f(m,n,\tau,\nu) &= \int_R p(t-mT_c)p^*(t-nT_c-\tau) \\ &\quad \times e^{i\pi[\alpha_1 t^2-\alpha_2(t-\tau)^2]}e^{i2\pi\nu t}\,dt.\end{aligned}$$

Thus we have that the cross ambiguity function of $s_1(t)$ and $s_2(t)$ can be written in terms of the coding sequences C_n and D_n and the function $f(m,n,\tau,\nu)$ as follows:

$$\chi_{s_1,s_2}(\tau,\nu) = \sum_{m=0}^{M-1}\sum_{n=0}^{M-1} C_m D_n^* f(m,n,\tau,\nu) \qquad (4)$$

Two important observations from above equation are:

1) Different chirp rates i.e. $\alpha_1,\ \alpha_2$ or different code sequences C_n and D_n, or different code lengths M, enable our waveforms to be diverse.
2) Proper selection of the code sequences C_m or D_m enable our waveforms to be orthogonal.

IV. Our Approach For Joint GMTI and SAR Processing

We will assume that the ground scene has both fast and slow movers. Hence, we will require a high PRF rate for exoclutter GMTI (i.e. to detect the fast mover from Doppler) and low PRF rate for SAR imaging and endoclutter GMTI. We will assume that we have two different transmit waveforms (SAR and Exoclutter GMTI) encoded with orthogonal codes. We will design the endoclutter GMTI/SAR waveforms to provide higher bandwidth than the exoclutter waveforms. Endoclutter GMTI/SAR pulses are coded with code C and exoclutter GMTI pulses are coded with code D. We will

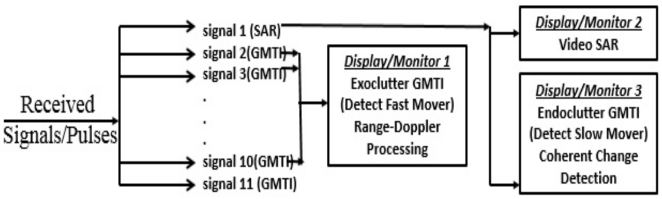

Fig. 1. Joint GMTI and SAR processing concept. We will transmit a group of 11 pulses at a time and repeat. Transmit pulse 1 is SAR pulse and coded with C. This waveform provides bandwidth of 600 MHz for high resolution SAR images and coherent change detection for endoclutter GMTI (to detect slow movers). PRF rate for SAR pulse is 300Hz. Transmit pulses 2-11 are exoclutter GMTI pulses and coded with D. These waveforms provide bandwidth of 200 MHz and PRF for these pulses is 1500Hz to detect the fast movers using Doppler

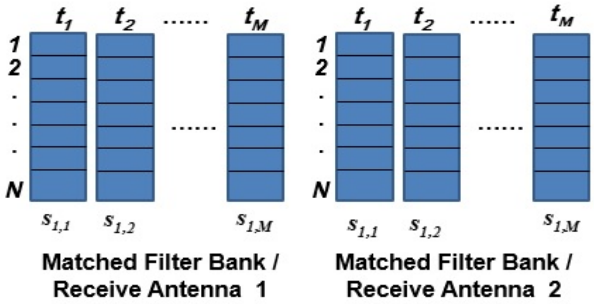

Fig. 2. Phase history data recording for endoclutter GMTI algorithm and SAR processing. $s_{1,1}$ is pulse 1 (SAR pulse) at time t_1; similarly, $s_{1,M}$ is pulse 1 at time t_M. We will perform DPCA based coherent change detection to detect the slow movers. Hence, we will record phase history in two receive antennas separated at 0.3 meter apart.

separate the SAR pulses by despreading it with the code C. More specifically, if a transmit pulse was coded with C, despreading with C will provide the highest auto-ambiguity response; but despreading with D will provide low cross-ambiguity response. In this manner, we can separate the SAR/endoclutter pulses from exoclutter pulses. We used the terminology *Multiplexed Matched Filter* to separate SAR and exoclutter GMTI pulses. Figure 1 illustrates our notional joint GMTI and SAR processing concept.

A. Endoclutter GMTI and SAR Image Processing

Figure 3 shows an algorithmic flow diagram for SAR imaging and endoclutter GMTI. For this type of GMTI [8] [9] [10] and SAR imaging, we will assume that a PRF rate of 1500 Hz is too high to detect slow movers and will cause unacceptable range ambiguity. Hence, we will have to set the PRF rate for SAR pulses to an appropriate level. In our example, we use a PRF rate of 300 Hz for SAR pulse (Pulse 1). To detect a slow mover, we form two SAR images from two different receivers that are displaced 0.3 meter apart (Displaced Phase Center Antenna, DPCA). Then perform coherent changed detection to detect the moving target. Figure 2 shows data recording for SAR imaging and endoclutter GMTI.

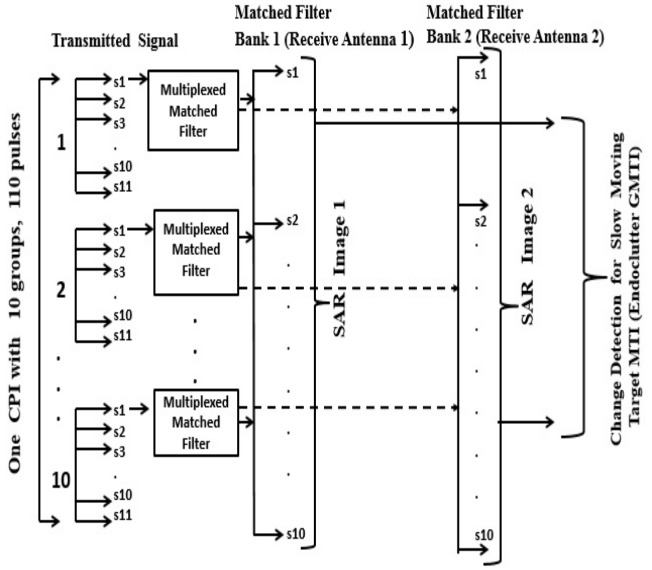

Fig. 3. Endoclutter GMTI algorithm and SAR image processing architecture. Out of 110 (10 repititions of 11 pulses) diverse, orthogonal waveforms for a CPI, we can extract 10 pulses (pulse 1 out of 11 pulses) using a multiples matched filter bank. These pulses (phase history data) can be recorded into two receivers at 0.3 meter apart (DPCA). Then form two SAR images from these two receivers and apply coherent change detection to detect the slow moving target

B. Exoclutter GMTI Processing

Figure 4 shows an algorithmic flow diagram for exoclutter GMTI. We will assume that a PRF rate of 1500 Hz will be sufficient for exoclutter GMTI. Hence, transmit pulses s_2 through s_{11} were sent at the PRF rate of 1500Hz. Consider a coherent processing interval (CPI) consists of 100 pulses. These 100 pulses can be obtained from 10 repetitions of 11 diverse, orthogonal waveforms. We have designated pulse number 1 (with high bandwidth) for SAR imaging. At the receiver, we will have a multiplexed matched filter to separate 10 (pulses 2 to 11) GMTI pulses and repeat it 10 times to accumulate 100 pulses for a CPI. Finally, apply FFT to develop a range-Doppler map of the moving object. The details of multiplexed matched filter concept can be found in [5]. Note that we can use all pulses (s_1 through s_{11}) for exoclutter GMTI to avoid potential issues of coherently combining several blocks of pulses in a CPI.

V. RESULTS

Based on our discussion on joint GMTI and SAR processing scheme, we develop scenarios to detect slow and fast moving targets simultaneously from an interrogated scene. Detecting slow moving targets (i.e. endoclutter targets) is a complex problem. Space-time adaptive processing and coherent change detection (CCD) are often used to detect slow moving targets. We have used DPCA-based (displaced phased center antenna) coherent changed detection to detect the slow movers. After detecting a moving target, smeared signature of this target can be focused using different algorithms [11], [14], [15], [9]. In this paper, we have not pursued to focus smeared energy of the moving targets.

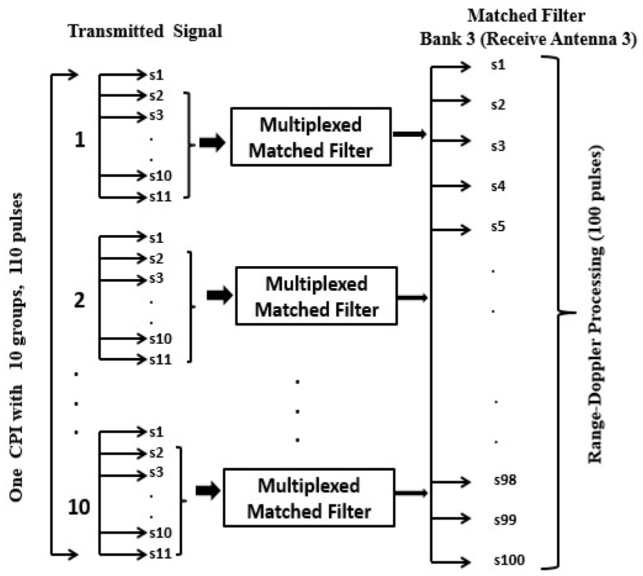

Fig. 4. Exoclutter GMTI algorithm concept. Out of 110 (10 repititions of 11 pulses) diverse, orthogonal waveforms for a CPI, we extract 100 pulses (10 repititions of pulses 2-11) at the receiver. Then apply range-Doppler processing.

A. *Endoclutter GMTI and SAR Processing Results*

Table I below presents key parameters used to develop signal processing algorithm for SAR image processing and endoclutter GMTI. The details of modeling targets (both stationary and moving) in synthetic aperture radar system can be found in different books. In this simulation, we have used moving target SAR signal processing theory presented in chapter 8 of the monograph by Soumekh [3]. In our first scenario, we had four stationary targets and one moving target in the scene. We generated SAR phase history data based on Table I parameters and recorded into two receive antennas separated by 0.3 meter. Two SAR images have been generated using backprojection algorithm (from two receive antennas). Because receive antennas were physically apart, the phase of the moving target signature will be slightly different but stationary target's phase will remain same. Hence, when we perform coherent change detection, moving targets signature will be present but stationary targets' signature will be cancelled. Notice that, non-coherent change detection (amplitude only change detection) will not reveal this phenomenology. Figures 5 and 6 show two SAR images constructed from two receive antennas' phase history data. Figure 7 shows detected slow moving target of the interrogated scene. In our second scenario, we had three stationary targets and two moving targets in the scene. Once again, we generated SAR phase history data based on Table I parameters and recorded into two receive antennas separated by 0.3 meter. Then two SAR images have been generated using backprojection algorithm (from two receive antennas). Figures 8 and 9 show two SAR images constructed from two receive antennas' phase history data. Figure 10 shows detected slow moving targets of the interrogated scene.

Parameters	Values
PRF	300 Hz
Bandwidth	600MHz
Radar Platform Velocity	75 m/sec
Distance Between Two Receivers	0.3 m
Carrier Frequency	16.9 GHz
Target Speed	5 m/sec

TABLE I
EXPERIMENTAL PARAMETERS USED TO MODEL AND DETECT SLOW MOVING TARGET (ENDOCLUTTER GMTI) PHENOMENOLOGY IN A SAR SYSTEM.

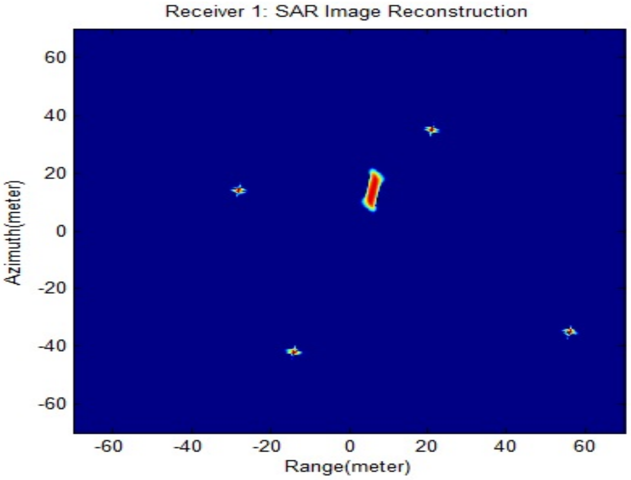

Fig. 5. SAR image formation from matched filter bank 1 (receive antenna 1). In this scenario, interrogated scene had 4 stationary point targets and one moving target. Moving target's signature smears in SAR imagery due to it's velocity

B. *Exoclutter GMTI Processing Results*

Table II below presents key parameters used to develop signal processing algorithm for fast moving target detection (exoclutter GMTI). Mentioned earlier, to detect fast movers based on Doppler, a high PRF rate is needed. Also, a low bandwidth is sufficient for exoclutter GMTI. Hence, PRF rate is set to 1500Hz and bandwidth is set to 200MHz to detect the fast moving targets. In our simulation, there were three moving targets in the scene. Two targets have the same velocity but situated in different locations; one target has different velocity than the other two. Figure 11 shows range-Doppler map of these three moving targets.

VI. DISCUSSION

Signal processing architecture for joint GMTI and SAR processing presented here assumes that transmit signals (endoclutter GMTI/SAR pulses and exoclutter GMTI pulses) are

Parameters	Values
PRF	1500 Hz
Bandwidth	200MHz
Radar Platform Velocity	75 m/sec
Carrier Frequency	16.9 GHz

TABLE II
EXPERIMENTAL PARAMETERS USED TO DETECT FAST MOVING TARGET (EXOCLUTTER GMTI).

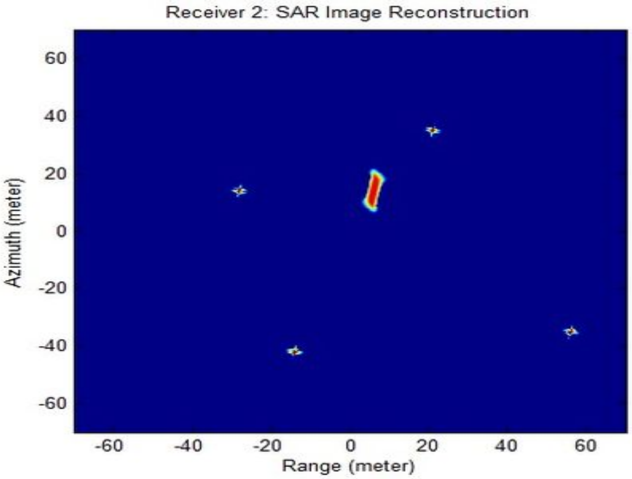

Fig. 6. SAR image formation from matched filter bank 2 (receive antenna 2). In this scenario, interrogated scene had 4 stationary point targets and one moving target. Moving target's signature smears in SAR imagery due to it's velocity

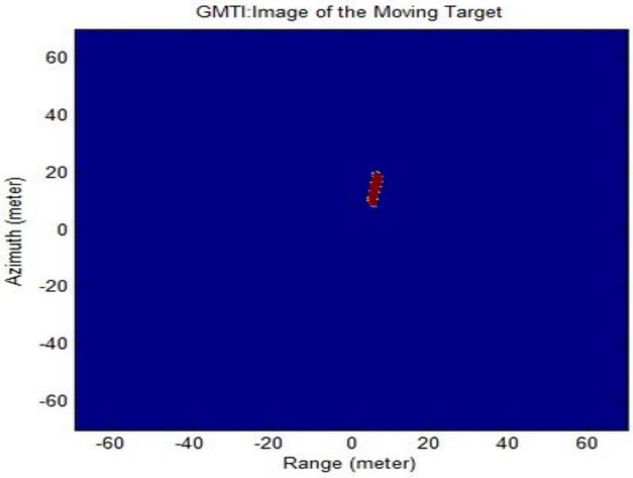

Fig. 7. Coherent change detection based on displaced phase center antenna (DPCA) has been used to detect the slow moving target from two SAR images presenetd in Figure 5 and Figure 6

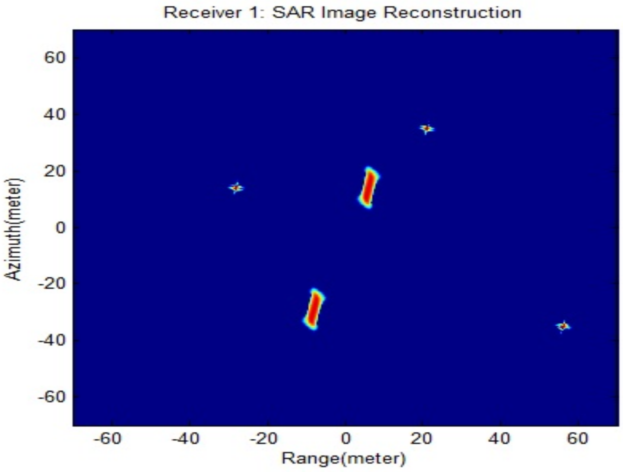

Fig. 8. SAR image formation from matched filter bank 1 (receive antenna 1). In this scenario, interrogated scene had 3 stationary point targets and two moving targets. Moving targets' signatures smear in SAR imagery due to their velocity.

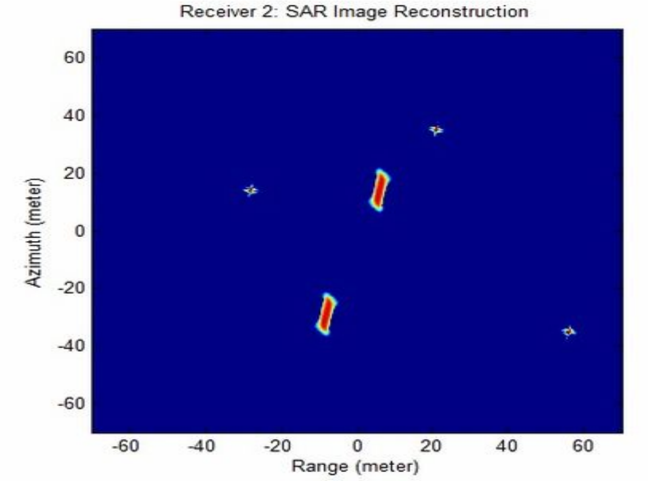

Fig. 9. SAR image formation from matched filter bank 2 (receive antenna 2). In this scenario, interrogated scene had 3 stationary point targets and two moving targets.

orthogonal and the system can separate them at the receiver (applying multiplex matched filter bank concept). In addition, PRF rate for exoclutter GMTI pulses (1500Hz) are much higher than the endoclutter GMTI / SAR pulses (300Hz) and the system is able to maintain it. Further, endoclutter GMTI pulses were designed to provide higher bandwidth (600MHz) than the exoclutter GMTI pulses (200MHz). Under this assumption, radar system can process the receive signals to produce three different outputs simultaneously: (1) Video SAR output and (2) Coherent change detection output for the slow moving targets detection (endoclutter GMTI) using low PRF SAR pulses, and (3) range-Doppler processing output for the fast moving target (exoclutter GMTI) using high PRF pulses. Several important aspects of joint GMTI and SAR design that we did not exmine yet are the followings: (1) One might argue that integrating M orthogonal SAR pulses to generate a SAR image will increase the SNR by a factor of $\sqrt{M}$. (2) As number of integrating pulses increases, computational costs will increase. However, this processing requirements are linear in time and could be accomplished in a massively parallel computer system (3) we can adapt the waveforms (PRF, Bandwidth) based on target/environment scenario (4) Do we incur polynomial time complexity to design particular orthogonal code? The answer to this question might be that we can use simple orthogonal code such as Walsh-Hadamard code (5) Will coded waveforms introduce Doppler tolerance issue? The answer to this question could depends on processing scheme. In SAR signal processing scheme, waveform's Doppler tolerance is not an issue because target has to move extremely fast (say mach 2 or above). In SAR images (regardless of the waveform), stationary target/scene is always localized; moving targets signature smear and displaced depending on the

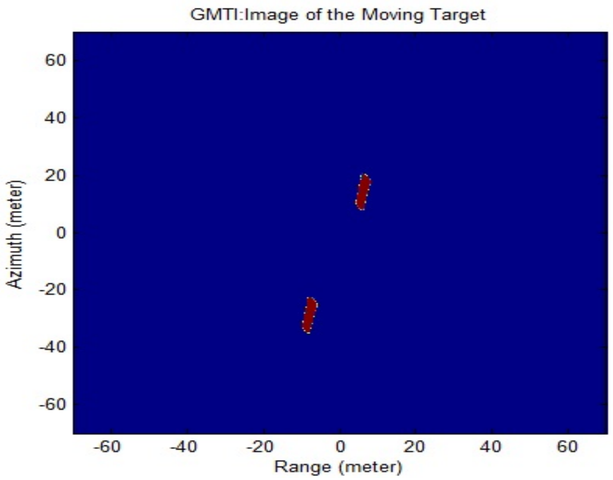

Fig. 10. Coherent change detection based on displaced phase center antenna (DPCA) has been used to detect the slow moving targets from two SAR images presenetd in Figure 8 and Figure 9

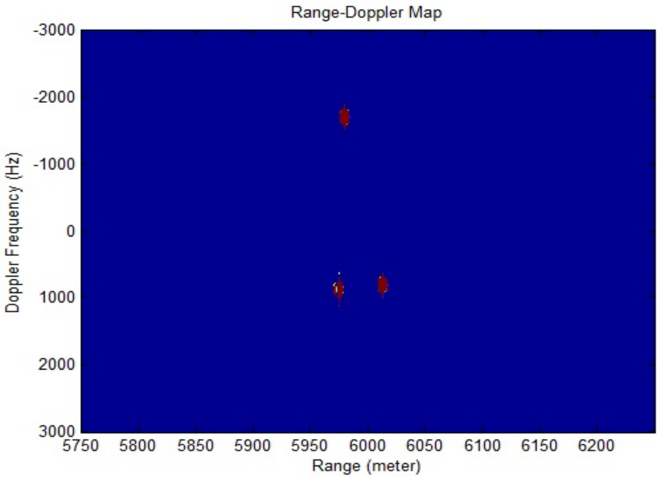

Fig. 11. Range-Doppler processing to detect fast moving targets. The interrogated scene has three moving targets. Two targets have same velocity; hence they generated same Doppler frequency.

velocity of the target. We need to correct phase history of the moving target (in SAR) and reprocess to properly geolocate the moving target. This was shown in a SAR system (e.g. AFRL's Gotcha Radar) that used LFM (Doppler tolerant) waveforms.

VII. CONCLUSION

In this research, we established a signal processing framework to accomplish joint GMTI and SAR exploitation reducing complexities associated with reconfiguring (i.e. setting beam pattern, PRF, bandwidth etc.) a radar system for the GMTI mode or SAR mode. Our approach allows efficient bandwidth utilization by employing appropriate bandwidths for GMTI pulses and SAR image formation pulses. Further, our approach solves range ambiguity issue associated with high PRF operation by using coded waveforms to separate individual waveforms.

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