Implementation of a Radar System Model and Detection Analysis

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***Abstract*--This paper depicts implementation of a radar system model and detection analysis by applying the Monte Carlo Simulation. For the radar system model development, we incorporated antenna geometry, transmit and receive signal model, antenna array and element pattern, thermal noise and clutter model. For detection analysis, we consider different scenarios such as fluctuating versus non fluctuating target, target located at different ranges, different clutter constants (gamma, γ), and different beta (β) values (clutter slope). For a given false alarm (P_{fa}), the simulation results show high Signal to Noise Ratio (SNR) requirements to detect a fluctuating target, target located at short range, beta>1 and higher clutter environment (gamma, γ).**

Keywords: Target detection, SNR, Probability of detection (Pd), Antenna, Thermal Noise, Clutter, Space-time adaptive processing

1. INTRODUCTION

Target detection, identification, and tracking are core functions of many radar systems. Radar research involves novel algorithm development, testing, and evaluation for target detection, identification, and tracking that could be implemented for a practical radar system. Availability of meaningful data for radar research is crucial. However, sometimes measured data format are not suitable for analysis quickly [2]. Hence, researchers develop radar system models that mimic the functionality of a practical radar system to generate synthetic data in a very simple format for research purposes. We developed an airborne radar model for synthetic data generation. Then, we used these data for detecting a target in different scenarios. Unless otherwise specified, Table 1 describes the simulation parameters that we have used for the radar model and target detection scenarios. The mathematical equations described here are taken from the EENG course notes [2] and [3].

TABLE 1.
Project Scenario Parameters

Variable	Value
M	128
N	11
P	2
f_0	1240 MHz
f_r	1984 Hz
τ	50 μs
P_t	1.5 kW
B	800 kHz
F_n (Noise Figure)	3 dB
N_c	361
h_a (aircraft altitude)	3073 m
v_a (aircraft velocity)	$\frac{d_x f_r}{2}$
R	66 km, 10km
γ	-3 dB
Array transit Gain	0 dB
Element Pattern	Cosine
d_x	0.10922 m
d_z	5.54 in
Transmit Taper	Uniform (None)
System Loses L_s	3 dB
Target ϕ	$0°$
Target θ	$0°$
Target ω	0.25

2. MODEL COMPONENTS

The principle components for radar system simulation are physical and antenna geometry, transmit and received signal model, antenna element and array pattern, thermal noise model, Jammer noise model, clutter model and target model [2].

2.1 ANTENNA ARRAY GEOMETRY

We considered a pulsed doppler airborne radar for velocity information. Antenna structure for this radar model is a planer array with N=11 azimuth elements and P=2 elevation elements. Azimuth and elevation spacing for the antenna array elements are d_x and d_z respectively. The top row and front element is the reference element. The signal first arrives at the reference element. The signals to the rest of the array elements are phased delayed from the reference element.

2.2 TRANSMIT AND RECEIVE SIGNAL

In this model, we consider the antenna array is a phased array and each antenna element has a transmit/receive module attached. Other antenna models can be simulated using phased array concepts. Transmitted signal can be modeled mathematically using the following equations [2]:

$$s(t) = a_t u(t) e^{j(\omega_0 t + \varphi)} \quad \textbf{[1]}$$

$$u(t) = \sum_{m=0}^{M-1} u_p(t - mT_r) \quad \textbf{[2]}$$

In the above equations, u(t) is the real value envelope function, ω_o is the carrier frequency, a_t is the transmitted pulse amplitude, M is the Coherent Processing Interval (CPI) and T_r is the Pulse Repetition Interval (PRI). The M pulses are used for Doppler filtering. The return signal from the m^{th} pulse is given by equation:

$$x_{mnp} = a_t e^{j2\pi(n v_x + p v_z)} e^{j2\pi m \varpi} \quad \textbf{[3]}$$

Where, v_x is the azimuth spatial frequency component given by the following equation:

$$v_x = \frac{d_x \cos\theta \sin\phi}{\lambda_o} \quad \textbf{[4]}$$

And, v_z is the elevation spatial frequency component given by the following equation:

$$v_z = \frac{d_z \sin\theta}{\lambda_o} \quad \textbf{[5]}$$

Different wave form models such as Linear Frequency Modulation (LFM) and Barker Code could be used for pulse compression gain. However, for this model simulation, we did not incorporate any pulse compression technique.

2.3 ANTENNA ELEMENT AND ARRAY PATTERN

The antenna element pattern we have used is a cosine pattern. The array transmit gain and element gain were 0dB. Element backlobe level was -30 dB. The 2D element pattern is defined as [2] following:

$$f(\theta,\phi) = \begin{cases} \cos\theta\cos\phi & -90^0 \le \phi,\theta \le 90^0 \\ b_e \cos\theta\cos\phi & 90^0 \ge \phi,\theta \ge 270^0 \end{cases}$$

Where, azimuth (ϕ) and elevation (θ) are in radar coordinates and b_e is the backlobe weighting factor. The spatial array radiation (voltage) and power patterns can be defined as [2] following:

$$F(\theta,\phi) = W(\theta,\phi) f(\theta,\phi)$$

$$G(\theta,\phi) = |W(\theta,\phi) f(\theta,\phi)|^2 = |W(\theta,\phi)|^2 g(\theta,\phi)$$

Where $g(\theta,\phi)$ is the power pattern and defined by $g(\theta,\phi) = |f(\theta,\phi)|^2$, $W(\theta,\phi)$ is the spatial array factor, $F(\theta,\phi)$ is the spatial array radiation and $G(\theta,\phi)$ is the power patterns.

2.4 NOISE MODEL

For the noise model, we used complex Gaussian random variables to model the noise of a worst case scenario. We assumed the receiver noise as white which means that the noise is mutually uncorrelated from pulse to pulse as well as element to element. This assumption is valid when Pulse Repetition Frequency (PRF) is much less than the waveform bandwidth and the bandwidth is also assumed to be much less than the transmit frequency.

The noise covariance matrix is given by the following equation:

$$R_n = \mathrm{E}\{\chi_n \chi_n^H\} \quad \textbf{[6]}$$

Which is simplified into the following equation:

$$R_n = \sigma^2 I_{MNP} \quad \textbf{[7]}$$

Thermal noise plays a very important role for interference suppression. Because the noise covariance matrix is full ranked, it is invertible. This is very important for space time adaptive processing. We can model Barrage Noise Jammer, however for this simulation we did not incorporate Jammer model for data generation.

2.5 CLUTTER MODEL

For the clutter model, we considered constant gamma (clutter constant of different environments) model for a city. We considered 361 clutter patches around the aircraft. We also computed ambiguous ranges and their grazing angles. We modeled the earth with a 4/3 effective radius. Angular extent $\Delta\phi$ of the clutter patch was calculated in the following equation:

$$\Delta\phi = \frac{2\pi}{N_c} \quad \textbf{[8]}$$

Elevation angle to a clutter patch is defined by the following equation:

$$\theta_c = -\sin^{-1}\left[\frac{R_c^2 + h_a(h_a + 2a_e)}{2R_c(a_e + h_a)}\right] \quad \textbf{[9]}$$

Where, a_e is the effective radius of the earth. The grazing angle, which is the angle between the line tangential to earth's surface at the clutter patch and a line extended from the airborne radar to the patch, is defined by the following equation:

$$\psi_c = -\sin^{-1}\left(\frac{R_c^2 - h_a(h_a + 2a_e)}{2R_c a_e}\right) \quad \textbf{[10]}$$

We calculated the horizon range and unambiguous range using the following equations:

$$R_h = \sqrt{h_a^2 + 2h_a a_e} \quad \textbf{[11]}$$

$$R_u = cT_r / 2 \quad \textbf{[12]}$$

where c is the speed of light. Clutter reflectivity is defined by the following equation:

$$\sigma_o(\theta_i, \phi_k) = \gamma \sin\psi_i \quad \textbf{[13]}$$

Using this equation, RCS of a clutter patch is computed at a particular azimuth and elevation using the following equation:

$$\sigma_{ik} = \sigma_o(\theta_i, \phi_k) R_i \Delta\phi \Delta R \sec\psi_i \quad \textbf{[14]}$$

Clutter to noise ratio at a single element is computed from the following equation:

$$\xi_{ik} = \frac{P_t G_t(\theta_i, \phi_k) g(\theta_i, \phi_k) \lambda^2 \sigma_{ik}}{(4\pi)^3 N_o B L_S R_i^4} \quad \textbf{[15]}$$

Finally, the clutter covariance matrix is computed from the following equations:

$$R_c = \mathrm{E}\{\chi_c \chi_c^H\} \quad \textbf{[16]}$$

$$R_c = \sigma^2 \sum_{i=0}^{N_r - 1} \sum_{i=0}^{N_c - 1} \xi_{ik} e(\upsilon_z) e^H(\upsilon_z) \otimes b(\varpi_{ik}) b^H(\varpi_{ik}) \otimes a(\upsilon_x) a^H(\upsilon_x) \quad \textbf{[17]}$$

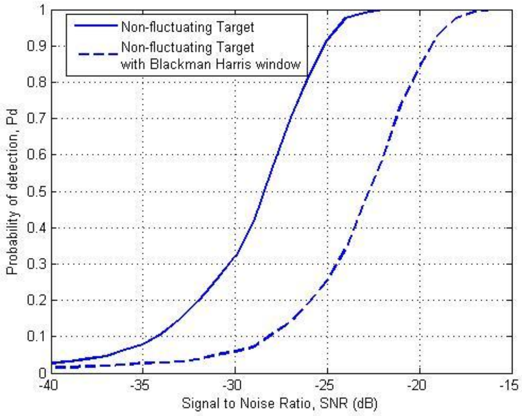

Figure 1: Probability of detection (Pd) vs. SNR for a non-fluctuating target located at 66km with $P_{fa} = 0.01$, $N_{trials} = 1000$, and Beta =1

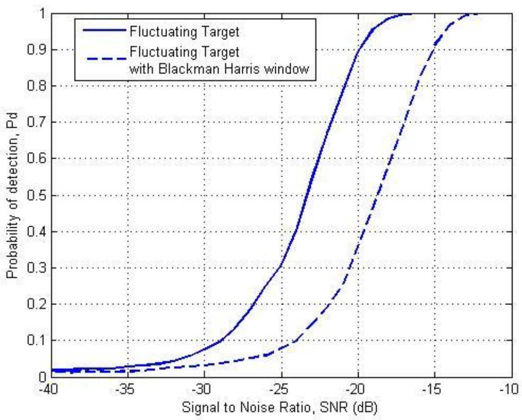

Figure 2: Probability of detection (Pd) vs. SNR for a fluctuating target located at 66km with $P_{fa} = 0.01$, $N_{trials} = 1000$, and Beta=1

where, σ^2 is N_oB and ϖ_{ik} is defined as follows in the following equation:

$$\varpi_{ik} = \frac{f_c(\theta_i, \phi_k)}{f_r} = f_c(\theta_i, \phi_k)T_r = \frac{2v_aT_r}{d_x}v_x \quad \textbf{[18]}$$

We also considered a narrow bandwidth assumption.

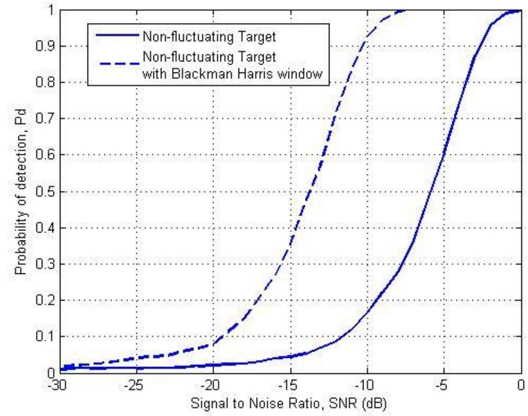

Figure 3: Probability of detection (Pd) vs. SNR for a non-fluctuating target located at 10km with $P_{fa} = 0.01$, $N_{trials} = 1000$, and Beta=1

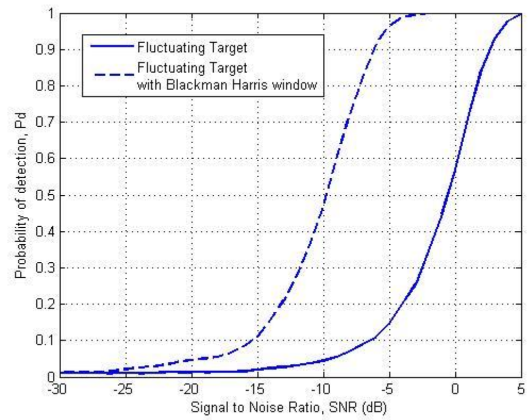

Figure 4: Probability of detection (Pd) vs. SNR for a fluctuating target located at 10km with $P_{fa} = 0.01$, $N_{trials} = 1000$, and Beta=1

3. TARGET MODEL

We considered a point target located at a certain range. In our simulation, we considered two different range locations of the point target. One location is at the range of 66km and another location is at the range of 10km.

4. DATA GENERATION

We can think of two different methods to generate data: snapshot-by-snapshot approach and block approach. Snapshot-by-snapshot approach is used if we want to build a single realization of one coherent

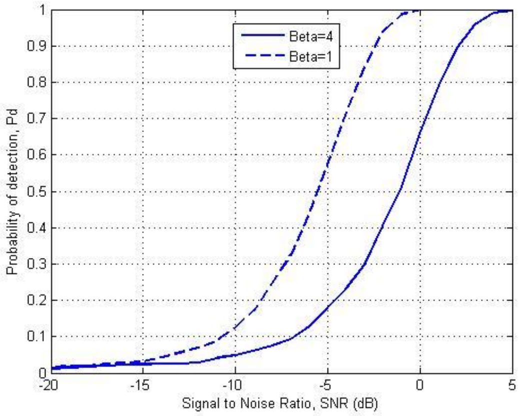

Figure 5: Probability of detection (Pd) vs. SNR for a non-fluctuating target located at 10km range with $P_{fa} = 0.01, N_{trials} = 1000$, and Beta=1,4

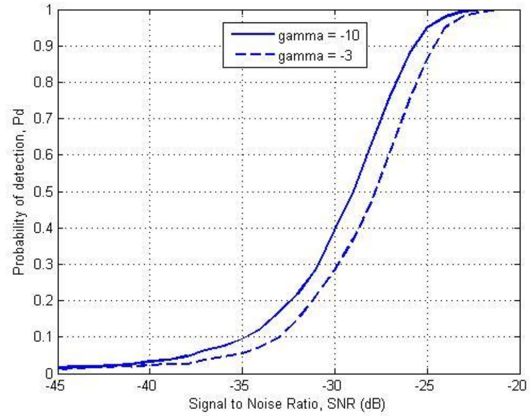

Figure 6: Probability of detection (Pd) vs. SNR for a non-fluctuating target located at 66km range with $P_{fa} = 0.01, N_{trials} = 1000$, and gamma =-3dB and -10dB. Gamma = -3dB corresponds to land clutter of Cities and gamma = -10dB corresponds to open woods at the frequency of 1.0 GHz. Skolnik [1]

processing interval (CPI) or a series of range cells in that CPI. On the other hand, block approach is used to build many realizations of one range cell. Block approach is suitable for SINR and probability of detection analysis and Monte Carlo simulation. We used block method to generate data for the target detection simulation.

5. RESULTS

Figures 1-6 show the results of the Monte Carlo Simulation of the target detection. For the fluctuating target model, we used Swerling case 2 model and placed the target at a range of 66km. In Swerling case 2, probability density function is exponential [1] and target fluctuations are independent from pulse to pulse. The radar cross section (RCS) of Swerling case 2 target is σ_{av}, which is the average over all values of target RCS. Therefore, to detect the small targets a large SNR is required. Figure 1 and Figure 2 show higher SNR requirements for fluctuating target at the range of 66km. In Figure 3 and Figure 4, for the fluctuating target located at 10km, a similar phenomena i.e. higher SNR requirement for target detection is observed. According to Skolnik [1], for a 0.5 probability of detection, the SNR loss factor is about 1.0 dB for a fluctuating target. Figure 1 and 2 shows that for a 0.5 probability of detection, this loss is about 3.0 dB.

For the non-fluctuating target located at the range of 10km, higher SNR is required. This is due to the fact that at the short range, clutter power impact is high. This contributes higher SNR requirements to detect a target. The impact of a high SNR requirement to detect a target at a short range is that the radar system has to change the threshold value when the target will be close to the radar platform.

The gamma (γ) value which was -3dB used for this simulation is considered to be suitable for strong clutter. According to Skolnik[1], this gamma value is used to model urban/city clutter at 1.0 GHz frequency.

For other clutter sources such as open woods has gamma value of -10dB. In our simulation, we incorporated very strong clutter to describe a clutter of city environment that has lots of buildings, metallic structures and roads made of concrete, etc. Figure 6 shows a SNR required to detect a target for the gamma (γ) of -3dB and -10 dB. With weak clutter such as for open woods, a low SNR is required to detect a target.
The Beta (β) value can be calculated using the following equation:

$$\beta = \frac{2v_a T_r}{d_x} \quad \textbf{[19]}$$

By using Table 1, the Beta value was calculated as 1.000034. The Beta value describes the slope of the clutter line. Beta=1 corresponds to filling the clutter space only once [3]. The following equation shows Beta>1 will introduce a higher clutter rank.

$$rank(R_c) \approx \lfloor N + (M-1)\beta \rfloor \qquad \textbf{[20]}$$

This will cause a doppler fold over and ambiguous doppler condition. For Beta=4, we observed a higher signal to noise ratio (SNR) requirements for target detection compared to Beta=1.

We also observed different SNR requirements for applying the Blackman Harris window in target doppler. For a target located at 66km, windowing requires higher SNR to detect a target while target located at 10km, windowing requires less SNR. Windowing suppress the sidelobes but widens the main beam. With a 0.25 target doppler, windowing allows more clutter power at the range of 66km and hence higher SNR is required to detect the target. On the other hand, with a 0.25 target doppler, at the range of 10km, windowing allows proportionately less clutter compare to 66km and hence low SNR is required to detect the target.

6. CONCLUSIONS

In this paper, the development of a radar system model was described. We described antenna geometry, transmit/receive signal model, antenna element, noise model and clutter model. We generated 1,000 realizations of a single range cell data using the block approach. We then used this data for a Monte Carlo simulation. We examined different SNR requirements to detect a target located in different ranges, different clutter environments, different clutter slopes (β) and Blackman Harris windowing. For a given false alarm (P_{fa}) of 0.01 and $N_{trials} = 1000$, the simulation results show high Signal to Noise Ratio (SNR) requirements to detect a fluctuating target, target located at short range, beta>1 and higher clutter environment (gamma, γ).

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